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INFLUENCE OF COMBINED PRESSURE GRADIENT AND TURBULENCE ON THE TRANSFER OF HEAT FROM A PLATE

A. R. BÜYÜKTÜR,* J. KESTIN† and P. F. MAEDER†
Brown University, Providence, Rhode Island

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Abstract—This paper describes the results of experiments on the combined effect of a favorable pressure gradient and free-stream turbulence intensity on the local transfer of heat from an isothermal flat plate to an air stream. Experiments were made with both laminar and turbulent boundary layers.

The experiments show that above about an intensity of turbulence of 1 per cent, the local rate of laminar convection is increased, the increase itself increasing with turbulence intensity. In contrast with a stagnation point, the increase is of modest magnitude, being of the order of 5–10 per cent for $Tu \approx 5$ per cent.

A turbulent boundary layer has turned out to be insensitive to variations in the free-stream turbulence intensity.

A series of traces taken with a hot-wire probe demonstrates that a laminar boundary layer carries in it quite intensive fluctuations of an obviously stable character. The appearance of higher frequencies and a slight increase in amplitude with increasing Reynolds number have been traced in a qualitative way.

NOMENCLATURE

- a, slope of dimensionless free-stream velocity distribution, see equation (1);
- Fr_L, Froessling number $(= Nu_L/Re_L^{1/2})$ based on idle distance;
- k, thermal conductivity;
- L, distance from hypothetical stagnation point to leading edge of plate ("idle distance");
- Nu_L, Nusselt number based on idle distance, see equation (4);
- Nu_x, local Nusselt number based on distance from tripping wire,
- Pr, Prandtl number;
- $\dot{q}$, heat flux;
- Re_L, Reynolds number based on idle length, see equation (5);
- Re_x, local Reynolds number based on distance from tripping wire;
- T, t, temperature;
- T_m, mean temperature, see equation (6);
- T_w, temperature at the wall;
- T_∞, temperature in free stream;
- Tu, longitudinal turbulence intensity;
- u, longitudinal velocity in boundary layer;
- U, free-stream velocity;
- U_x, free-stream velocity at distance x from leading edge of plate;
- U_0, free-stream velocity at leading edge of plate;
- x, length co-ordinate;
- ϵ, emissivity;
- η, Blasius variable $[= \frac{1}{2} y\sqrt{(U_0/\nu x)}]$;
- ν, kinematic viscosity;
- ξ, distance from stagnation point.

1. INTRODUCTION

IN A PREVIOUS communication [1] there were given preliminary results regarding the combined effect of a favorable pressure gradient and increased free-stream turbulence on the local rate of heat transfer from a flat plate. Owing to the inconclusive nature of those results, and owing to their lack of completeness, we have performed additional, and more careful measurements with a similar arrangement. Since the effects revealed by the present investigation were of a much reduced magnitude compared with those observed earlier in a preliminary way, great care was taken to repeat them several times in

* Research Associate.
† Professor of Engineering.

order to acquire the necessary degree of confidence. The experimental arrangement was dismantled and re-assembled several times, and maximum care was taken to secure good reproducibility and precision. The latter is evidenced by a very low degree of scatter in the final results.

Accordingly, the present paper will describe experiments on the combined effect of a favorable pressure gradient and varying free-stream turbulence intensity on the local transfer of heat from a flat plate across a laminar, and later, across a tripped, turbulent boundary layer.

2. EXPERIMENTAL ARRANGEMENT

All measurements were carried out in the Brown University 22 in × 32 in subsonic wind tunnel. It was the same tunnel as was used in [1], except that its circuit had been closed. On closing, an air cooler was added to the wind tunnel circuit, and its presence greatly improved the precision of the control of temperature in the free-stream air.

The transfer of heat was measured essentially in the same plate as that described in [1], except that it was completely reconditioned and placed against the wall of the test section instead of in the center, as indicated in Fig. 1. A suitable amount of suction insured that the boundary layer started at the leading edge *a*. The suction rate was controlled by the adjustable vent *b*. A favorable pressure gradient was imposed by mounting the auxiliary wall *C*.

The two local heaters h_1 and h_2 were re-built and allowed measurements of local coefficients of heat transfer to be made (actually over a nomimal area 10 in × 0·5 in, giving an effective working area of 35·67 cm²) with a precision estimated to be of the order of ±1·5 per cent at low Nusselt numbers increasing to ±1·0 per cent at high Nusselt numbers. The corresponding Reynolds number could be determined with an error not exceeding ±2 per cent and 1 per cent, respectively.

In order to eliminate the need to re-polish the surface and to keep track of its changing emissivity, the surface of the strip heater was painted black, providing a constant emissivity of $\epsilon = 0{\cdot}97$. Thus the radiation correction could be determined very precisely.

At low rates of heat transfer, that is at low Reynolds numbers, the radiation correction amounted to as much as 50 per cent of the heat transfer rate. Nevertheless, this correction was known very accurately, and the preceding estimates take this circumstance into account. As a final check, preliminary measurements were carried out on a flat plate at zero pressure gradient, i.e. with the auxiliary wall *C* in Fig. 1 removed, and the results did not differ from Pohlhausen's theoretical solution by more than 1·5 per cent in the Nusselt number even at the lowest Reynolds numbers.

By adding an extra steam jacket at *d* it was possible to improve the temperature distribution on the surface of the plate in the streamwise direction. A typical temperature profile along the flow direction is shown in Fig. 2. This reduced the correction for temperature steps.

The electrical power input to the metered heater (h_1 or h_2, as occasion demanded) was supplied by a transistorized d.c. power supply unit which was regularly calibrated and checked by means of a digital voltmeter and a standard resistance. The second unit heater was supplied with a.c. current to bring its surface temperature

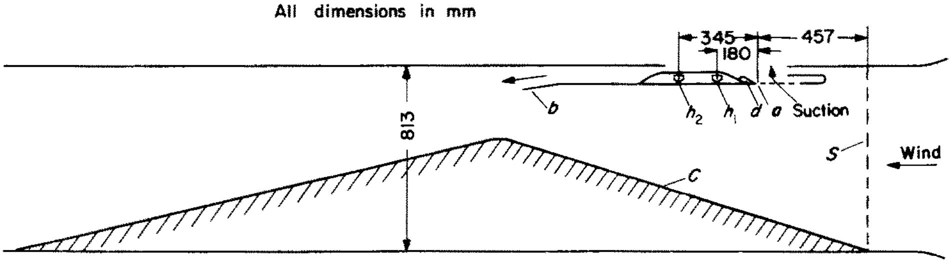

FIG. 1. Schematic diagram and experimental arrangement.

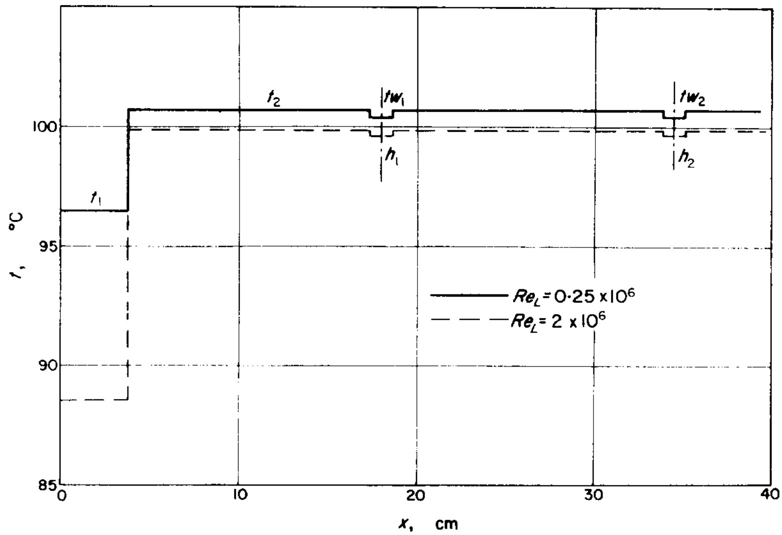

FIG. 2. Typical temperature profile in the flow direction at center of plate.

up to the required level. Great care was taken to maintain the temperature inside the unit heater equal to that of the steam in the jacket (to within 0·1 °C at the highest Reynolds number, decreasing to 0·05°C at lower Reynolds numbers) thus obviating the need to apply any corrections for heat leaks into the interior of the plate.

The turbulence intensity of the free tunnel, or that increased by the insertion of screen S in Fig. 1, was measured with the aid of a highly sensitive, transistorized hot-wire anemometer. The values of turbulence intensity quoted later always refer to the free stream exactly opposite the unit heater and a distance of 1·5 in away from it in a transverse direction.

3. EXPERIMENTAL RESULTS

3.1 *External flow*

The velocity in the free stream was determined with the aid of a Pitot-static tube at a transverse distance of 1·5 in, and the results are shown in Fig. 3. It is seen that the free-stream velocity can be assumed to have increased linearly, the slope being independent of wind speed. Using least squares, it was found that

$$a = \frac{\mathrm{d}[U(x)/U_0]}{\mathrm{d}x} = 5{\cdot}824 \times 10^{-3} \text{ 1/cm.} \quad (1)$$

Thus the flow over the plate corresponded to that which would exist in Hiemenz flow with the boundary layer starting at an idle distance

$$L = 171{\cdot}7 \text{ cm} \quad (2)$$

from the stagnation point, as illustrated in Fig. 4. The free-stream velocity was of the form

$$\frac{U(x)}{U_0} = 1 + \frac{x}{L}, \quad (3)$$

with L given in (2). This idle length L provides a natural scaling factor for the Nusselt number

$$Nu_L = \frac{\dot{q}L}{k} \quad (4)$$

and for the Reynolds number

$$Re_L = \frac{U_0 L}{\nu}. \quad (5)$$

The values of the thermal conductivity, k, and kinematic viscosity, ν, were taken at the mean temperature

$$T_m = \tfrac{1}{2}(T_w + T_\infty). \quad (6)$$

The density, ρ, was evaluated at the free-stream conditions.

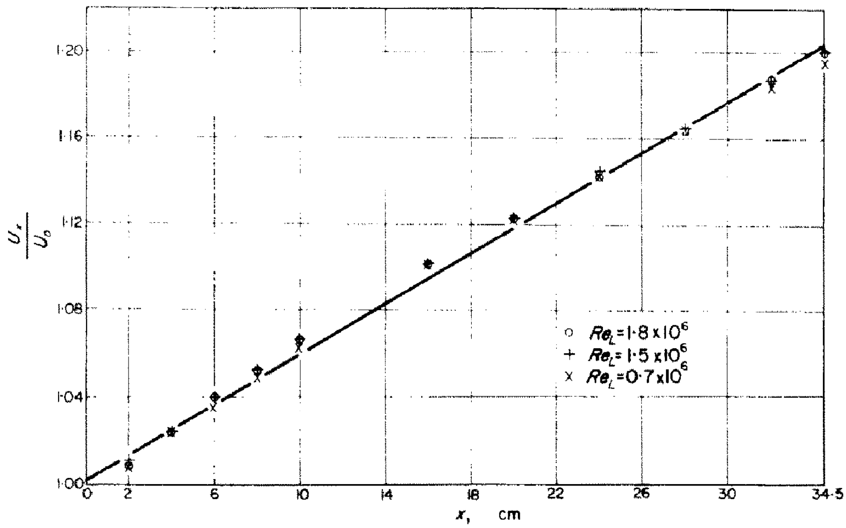

FIG. 3. Velocity distribution in the free stream along the plate.

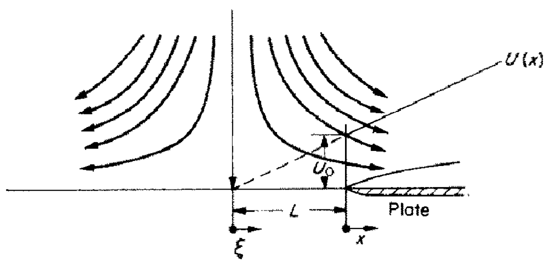

FIG. 4. Flow model. Hiemenz flow with idle starting length L.

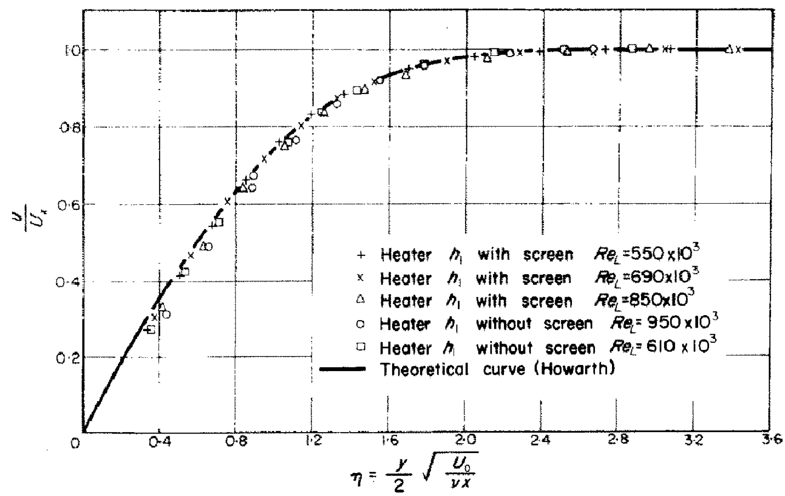

FIG. 5. Velocity profiles in the laminar boundary layer; heater h_1.

3.2 *Velocity profiles*

Several typical laminar boundary layer velocity profiles are seen plotted in Figs. 5 and 6. They have been compared with the theoretical calculations provided by Howarth [2] for a free-stream velocity of the form of (3). It is noted that Howarth provided a solution in the form of a series expansion in the parameter x/L with coefficients computed for terms up to $(x/L)^6$, and the good agreement displayed in the figures testifies to the suitability of the idealized flow model adopted for further analysis. The systematic deviation for the second heater h_2 in Fig. 6 is attributed to the fact that Howarth's series solution does not converge too well for $x/L = 0{\cdot}2009$ at h_2; the convergence is superior at $x/L = 0{\cdot}1048$ at h_1. It was estimated by Howarth that the seven terms in the series for $u(x, y)$ assured apparent convergence for $x/L \leqslant 0{\cdot}125$.

Heat transfer measurements were also made with the boundary layer tripped at the leading edge with the aid of rough emery and a tripping

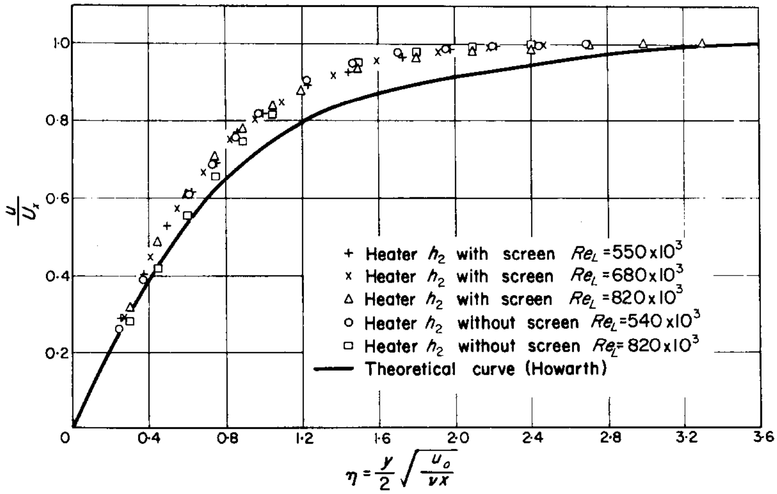

FIG. 6. Velocity profiles in the laminar boundary layer; heater h_2.

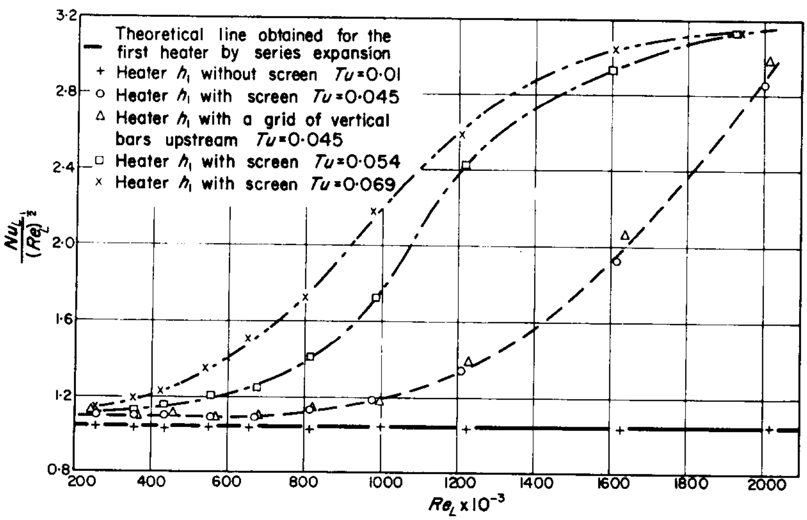

FIG. 7. Heat transfer across untripped boundary layer; heater h_1, $x/L = 0{\cdot}1048$.

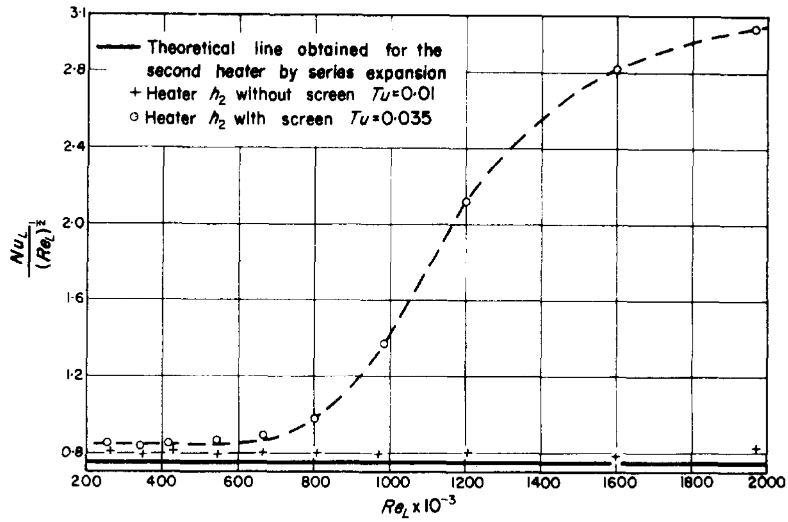

FIG. 8. Heat transfer across untripped boundary layer; heater h_2, $x/L = 0{\cdot}2009$.

rod. The turbulent nature of the resulting boundary layers was ascertained by observing the signal from the hot wire when placed inside the boundary layer. The signal was displayed on a cathode-ray oscilloscope, thus making it unnecessary to survey the mean velocity profile in the turbulent boundary layer.

3.3 *Laminar heat transfer*

The results of the heat transfer experiments with an untripped boundary layer are shown plotted in Figs. 7 and 8; they have also been listed in Table 1. The diagram of Fig. 7 refers to heater h_1 at $x/L = 0{\cdot}1048$, whereas Fig. 8 refers to heater h_2 at $x/L = 0{\cdot}2009$.

It is easy to show that in the laminar range, the Nusselt number $Nu_L = hL/k$ must be proportional to $Re_L^{1/2} = (U_0L/\nu)^{1/2}$, so that the group

$$Fr_L = \frac{Nu_L}{(Re_L)^{1/2}} \tag{5}$$

turns out to be a function of the Prandtl number, Pr, and of the relative abscissa x/L only; thus

$$Fr_L = Fr_L\,(Pr, x/L). \tag{6}$$

We propose the name *Froessling number* for the group Fr, because the group seems to have appeared explicitly for the first time in reference [3].

On the assumption that there is no effect from varying free-stream turbulence, a plot of the Froessling number Fr_L against the Reynolds number Re_L at a constant Prandtl number should show a constant value for a constant value of x/L, i.e. a constant value separately for each of the two heaters. These constant values have been computed by the method outlined in the companion reference [4]. Thus

$$\left.\begin{aligned} Fr_L &= 1{\cdot}047 \quad \text{at } x/L = 0{\cdot}1048 \text{ for } h_1 \\ Fr_L &= 0{\cdot}743 \quad \text{at } x/L = 0{\cdot}2009 \text{ for } h_2 \end{aligned}\right\}. \tag{7}^*$$

Departures from a constant value must be expected on two counts. First, as the Reynolds number Re_L increases, there will be transition to turbulent flow, the transition Reynolds number advancing with increasing intensity of turbulence. Secondly, even before transition sets it, there may be a change due to varying intensity of turbulence, Tu. The diagrams show both these effects, and as a consequence, the relation in (6) must be modified to include the intensity of turbulence as an independent variable. Hence we write

$$Fr_L = Fr_L(Pr, x/L, Tu). \tag{8}$$

An examination of Fig. 7 shows that the

* The value for $x/L = 0{\cdot}2009$ ($Fr_L = 0{\cdot}743$) is doubtful because the series ceases to converge here.

Table 1. Experimental results: untripped boundary layer

Air speed U_∞, m/s	Temp. difference ΔT, °C	Measured power, W	Radiation correction Pr, W	Correction for non-isothermal condition, C	Corrected Nusselt number Nu_L	Reynolds number, $Re_L \times 10^{-3}$	Froessling number, Fr_L
			Heater h_1 without screen; $Tu = 0{\cdot}01$				
3·14	76·73	4·7470	2·3353	0·9940	526·34	256·33	1·0396
4·37	76·65	5·1732	2·3357	0·9980	617·14	356·70	1·0333
5·33	76·47	5·4233	2·3321	0·9973	674·20	434·20	1·0231
6·79	76·28	5·8520	2·3283	0·9985	769·43	552·95	1·0347
8·08	76·47	6·1726	2·3317	0·9979	837·31	658·53	1·0318
10·00	76·43	6·6258	2·3321	1·0033	931·39	816·15	1·0309
12·32	76·67	7·1805	2·3379	1·0044	1046·12	1003·76	1·0441
15·07	77·07	7·6715	2·3443	1·0012	1149·37	1228·20	1·0371
20·03	76·43	8·4842	2·3315	1·0081	1328·23	1631·21	1·0399
24·65	75·87	9·0487	2·3117	0·9961	1483·11	2022·04	1·0429
			Heater h_1 with screen; $Tu = 0{\cdot}045$				
3·11	76·88	4·8930	2·3403	0·9970	554·36	254·06	1·0998
4·39	76·84	5·3625	2·3389	0·9952	658·01	375·97	1·0997
5·33	76·74	5·5900	2·3373	0·9974	723·58	434·68	1·0974
6·82	77·31	6·1453	2·3494	0·9990	818·55	557·05	1·0967
8·22	77·31	6·4782	3·3466	0·9932	896·34	671·76	1·0936
10·02	76·89	7·0686	2·3421	0·9948	1028·84	817·38	1·1379
12·02	76·56	7·7616	2·3375	0·9977	1182·42	979·61	1·1946
14·88	76·46	9·1565	2·3366	1·0027	1480·47	1212·47	1·3445
19·82	76·05	13·6850	2·3173	1·0172	2444·87	1614·60	1·9240
24·40	77·90	21·6700	2·3476	1·0195	4002·78	2001·53	2·8659
			Heater h_1 with a grid of vertical bars upstream $Tu = 0{\cdot}045$				
2·94	78·40	4·9841	2·3808	1·0038	551·07	240·06	1·1247
4·37	77·97	5·4589	2·3684	0·9974	661·91	356·90	1·1079
5·46	77·95	5·8384	2·3662	0·9948	746·00	446·00	1·1170
6·92	77·84	6·1832	2·3630	0·9976	819·60	565·66	1·0897
8·29	77·67	6·5866	2·3608	0·9964	909·22	676·40	1·1055
10·07	77·10	7·1423	2·3486	0·9954	1039·30	820·34	1·1474
12·21	76·68	7·8069	2·3369	1·0068	1178·90	994·91	1·1819
15·10	76·93	9·5311	2·3403	1·0046	1548·75	1231·32	1·3957
20·07	76·45	14·5483	2·3269	1·0055	2645·84	1635·50	2·0698
24·49	78·11	22·3500	2·3526	1·0065	4249·24	2009·90	2·9972
			Heater h_1 with screen; $Tu = 0{\cdot}054$				
3·12	79·01	4·8688	2·3896	0·9945	526·33	255·53	1·0412
4·30	78·78	5·5316	2·3848	0·9937	670·36	352·20	1·1295
5·30	78·27	5·9240	2·3717	0·9934	761·68	433·86	1·1563
6·83	78·30	6·6002	2·3742	0·9978	901·65	558·59	1·2064
8·25	78·18	7·2050	2·3712	0·9989	1031·60	674·86	1·2557
10·02	76·99	8·2390	2·3427	0·9982	1277·22	816·47	1·4135
12·09	76·14	10·1791	2·3292	1·0030	1710·36	983·90	1·7242
14·91	77·12	14·8336	2·3407	1·0043	2687·41	1217·56	2·4354
19·57	76·45	19·4336	2·3172	1·0038	3717·51	1599·06	2·9398
23·65	76·48	22·4000	2·3176	1·0048	4355·39	1932·49	3·1330

Table 1—continued

Air speed U_∞, m/s	Temp. difference ΔT, °C	Measured power, W	Radiation correction Pr, W	Correction for non-isothermal condition, C	Corrected Nusselt number, Nu_L	Reynolds number, $Re_L \times 10^{-3}$	Froessling number, Fr_L
Heater h_1 with screen; $Tu = 0.069$							
3·11	78·30	5·0664	2·3733	0·9974	574·92	254·25	1·1401
4·31	77·93	5·6477	2·3623	0·9938	707·09	352·07	1·1916
5·21	77·77	6·0606	2·3591	0·9928	799·01	425·50	1·2249
6·64	77·12	6·9250	2·3409	0·9947	995·59	542·45	1·3517
8·03	76·83	7·9407	2·3335	0·9954	1221·19	655·28	1·5085
9·91	75·34	9·3380	2·3018	1·0016	1550·21	805·83	1·7269
11·95	76·81	12·3158	2·3315	1·0028	2159·37	975·72	2·1860
14·77	76·80	15·4680	2·3295	1·0031	2841·86	1206·62	2·5871
19·70	76·14	19·4550	2·3062	1·0042	3738·39	1610·35	2·9459
23·74	76·25	22·3875	2·3048	1·0057	4368·01	1942·95	3·1336
Heater h_2 without screen; $Tu = 0.01$							
3·57	77·85	4·3332	2·3676	1·0043	418·65	267·80	0·8089
4·68	77·71	4·5702	2·3638	1·0029	471·42	351·43	0·7953
5·72	77·29	4·8490	2·3632	0·9989	532·87	429·75	0·8128
7·29	77·48	5·0928	2·3540	0·9997	588·76	547·54	0·7956
8·87	77·53	5·4170	2·3610	1·0003	656·17	665·95	0·8040
10·83	76·39	5·6750	2·3323	0·9999	727·61	810·72	0·8081
12·97	77·11	6·0042	2·3486	0·9993	789·35	972·43	0·8004
16·12	76·97	6·4362	2·3447	0·9981	886·04	1208·23	0·8060
21·39	76·78	6·9430	2·3403	1·0010	996·24	1602·27	0·7870
26·26	77·69	7·8091	2·3580	1·0004	1168·49	1972·77	0·8319
Heater h_2 with screen; $Tu = 0.035$							
3·42	76·94	4·3357	2·3464	0·9996	430·25	256·50	0·8495
4·55	76·74	4·5702	2·3401	0·9962	485·21	341·12	0·8307
5·54	76·71	4·8451	2·3403	0·9960	545·15	415·27	0·8459
7·21	77·77	5·3232	2·3612	0·9956	637·32	541·96	0·8657
8·85	77·87	5·7550	2·3620	0·9938	730·64	665·27	0·8957
10·70	77·31	6·4392	2·3500	0·9945	885·60	802·93	0·9883
13·13	76·89	8·6119	2·3383	0·9930	1368·01	985·28	1·3781
16·01	76·67	13 0850	2·3291	0·9989	2339·00	1201·61	2·1337
21·29	76·21	18·6368	2·3155	1·0004	3565·20	1597·99	2·8203
26·02	78·16	22·3132	2·3536	1·0013	4260·57	1965·74	3·0388

measurements on the first heater at $Tu = 0{\cdot}01$ agree very well with the essentially zero-intensity-of-turbulence value (7) and show no evidence of transition up to $Re_L = 2 \times 10^6$. As the turbulence intensity is increased to $Tu = 0{\cdot}045$, the value of the Froessling number increases to

$$Fr_T = 1{\cdot}097, \quad \text{i.e. by } 6{\cdot}2 \text{ per cent;}$$

evidence of incipient transition appears at $Re_L = 0{\cdot}8 \times 10^6$ approximately.

In order to show that this is purely an effect of intensity and not scale of turbulence, the same intensity of turbulence was obtained with a row of vertical bars in addition to the usual screen. It is seen that the difference in the results is insignificant.

The same behavior is reproduced for the higher intensities of turbulence, and for heater h_2. Thus, the variation of the Froessling number for laminar flow with intensity of turbulence appears to be systematic, as evidenced by the

Table 2. Experimental results; tripped boundary layer (turbulent)

Air speed U_∞, m/s	Temp. difference ΔT, °C	Measured power, W	Radiation correction P_r, W	Correction for nonisothermal condition, C	Corrected Nusselt number, Nu_x	Reynolds number, $Re_x \times 10^{-3}$
			Heater h_1 without screen; turbulent boundary layer			
3·97	75·92	7·8125	2·3095	0·9999	117·40	34·80
6·31	75·45	10·0670	2·2938	0·9995	167·08	55·30
8·97	75·61	12·7330	2·2938	1·0002	223·50	78·70
14·86	74·85	17·4760	2·2680	0·9998	325·00	130·25
23·73	74·98	24·6760	2·2655	1·0010	479·00	209·00
			Heater h_1 with screen; turbulent boundary layer			
3·99	76·37	7·8140	2·3148	1·0022	116·43	35·02
6·30	76·25	10·2000	2·3084	0·9993	167·91	55·40
8·95	75·97	12·7950	2·3008	0·9993	223·65	78·70
14·78	75·13	17·5290	2·2738	0·9999	329·13	130·00
23·44	75·53	24·7162	2·2818	1·0020	480·53	206·50
			Heater h_2 without screen; turbulent boundary layer			
4·28	76·32	6·433	2·339	0·9991	172·79	74·64
6·81	76·00	8·029	2·309	0·9982	242·73	118·65
9·71	75·60	10·170	2·297	0·9973	336·18	169·24
15·99	75·18	14·778	2·286	0·9991	535·24	278·42
25·57	75·35	21·212	2·282	0·9996	809·70	446·33
			Heater h_2 with screen; turbulent boundary layer			
4·32	75·88	6·366	2·308	0·9975	172·54	75·30
6·86	75·64	8·036	2·300	0·9975	244·71	119·47
9·72	75·24	10·245	2·288	0·9977	341·26	169·30
16·00	75·11	14·792	2·280	0·9975	537·82	278·91
25·53	75·19	21·253	2·276	1·0000	813·40	445·63

diagram in Fig. 9. The diagram shows the relative increase in the rate of heat transfer in the form of the ratio

$$\frac{Fr_L(Tu)}{Fr\,(0)} = \frac{Nu_L(Tu)}{Nu_L(0)}$$

of the actual rate to that at zero intensity of turbulence under identical conditions as a function of the turbulence intensity *Tu*.

The increase in the local rate of heat transfer has turned out to be modest in size and much smaller than that discovered earlier, [6], in the neighbourhood of the stagnation line on a cylinder in cross-flow. The present measurements should be regarded as superseding the very preliminary measurements involving a pressure gradient (but *not* those reported for zero pressure gradient) in [1].

3.4 *Signal from boundary layer*

In order to investigate the character of the flow in the boundary layer, a hot-wire probe was placed opposite the center-line of heater h_1 at a distance of 0·5 mm from the wall (corresponding to 0·2 of the boundary layer thickness), and traces were displayed from it on an oscilloscope. These are reproduced as Figs. 10–21; a key to these traces is contained in Fig. 22.

The traces in Figs. 10 and 11 have been taken at $Tu = 0{\cdot}01$ and $Re_L = 0{\cdot}25 \times 10^6$ and 2×10^6

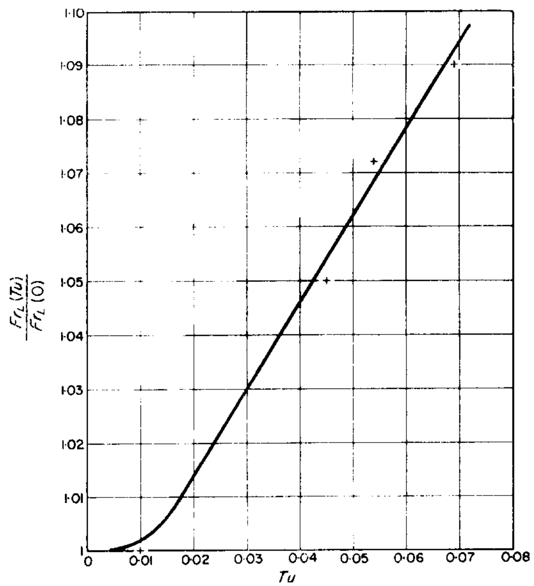

FIG. 9. Increase of rate of laminar heat transfer with turbulence intensity. Plot of $Fr_L(Tu)/Fr_L(0) = Nu_L(Tu)/Nu_L(0)$ against turbulence intensity Tu.

respectively. They serve principally to define the sensitivity of the oscilloscope setting; it is noted that a waviness just makes its appearance at $Re_L = 2 \times 10^6$.

The next series of ten traces has been taken with $Tu = 0{\cdot}045$ but at increasing Reynolds numbers.

It is seen that a fairly regular Tollmien–Schlichting wave makes its appearance as early as at $Re_L = 0{\cdot}25 \times 10^6$. The wave grows in amplitude and loses its regular shape, first at nearly constant amplitude and later with increasing amplitude and frequency. The traces exhibit clearly the so-called cascading of eddies. Finally, at $Re_L = 2 \times 10^6$ the signal is nearly that for fully developed turbulent flow. Thus the pictures confirm our previous interpretation regarding the transitional nature of the flow associated with the ascending branches of the curves in Figs. 7 and 8. The pictures also confirm the view that the flow in a laminar boundary layer carries an oscillating component in the presence of a turbulent free stream. This phenomenon was clearly anticipated in earlier communications, e.g. [5–9].

The pictures of the traces in Figs. 12–21 show that turbulent flow in a boundary layer is associated with a previous breakdown in stability. Consequently, laminar flow need not be associated with a complete absence of disturbances. Indeed, in the presence of constant excitation from the free-stream turbulence, the laminar boundary layer carries more-or-less regular oscillations, which, however, *are stable* ones.

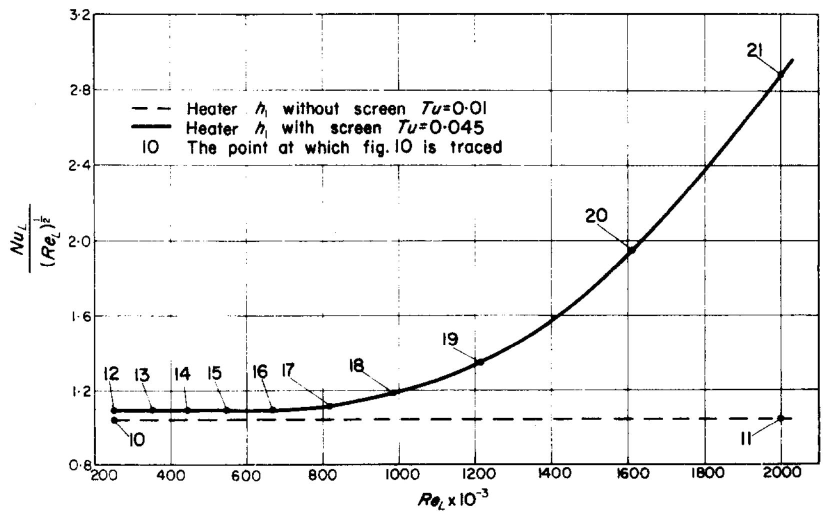

FIG. 22. Key to Figs. 10–21.

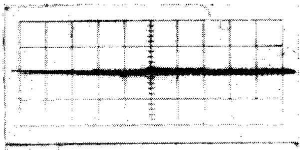

FIG. 10. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}25 \times 10^6$; $Tu = 0{\cdot}01$.

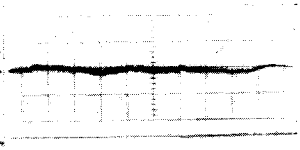

FIG. 11. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 2 \times 10^6$; $Tu = 0{\cdot}01$.

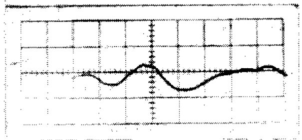

FIG. 12. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}25 \times 10^6$; $Tu = 0{\cdot}045$.

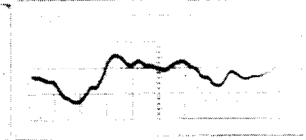

FIG. 13. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}35 \times 10^6$; $Tu = 0{\cdot}045$.

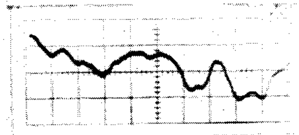

FIG. 14. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}45 \times 10^6$; $Tu = 0{\cdot}045$.

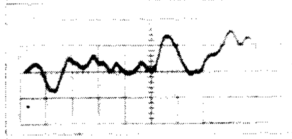

FIG. 15. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}55 \times 10^6$; $Tu = 0{\cdot}045$.

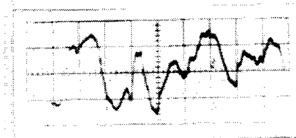

FIG. 16. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}675 \times 10^6$; $Tu = 0{\cdot}045$.

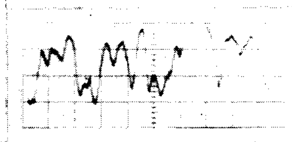

FIG. 17. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 0{\cdot}80 \times 10^6$; $Tu = 0{\cdot}045$.

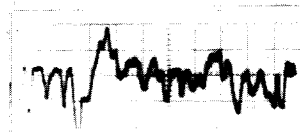

FIG. 18. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 1{\cdot}0 \times 10^6$; $Tu = 0{\cdot}045$.

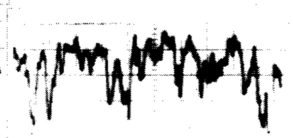

Fig. 19. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 1{\cdot}2 \times 10^6$; $Tu = 0{\cdot}045$.

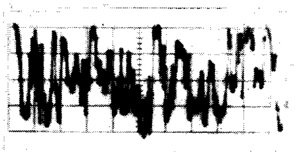

FIG. 20. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 1{\cdot}6 \times 10^6$; $Tu = 0{\cdot}045$.

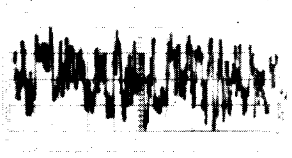

FIG. 21. Oscilloscope trace at $0{\cdot}2\ \delta$; $Re_L = 2{\cdot}0 \times 10^6$; $Tu = 0{\cdot}045$.

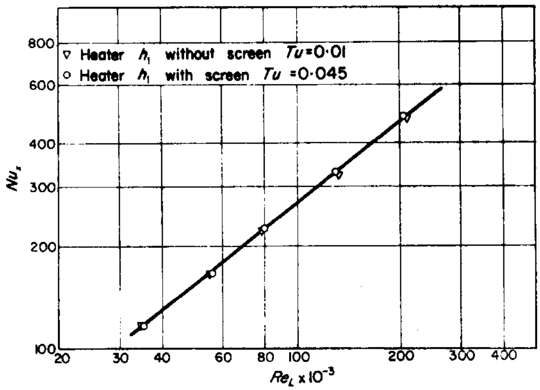

FIG. 23. Heat transfer from heater h_1; turbulent boundary layer.

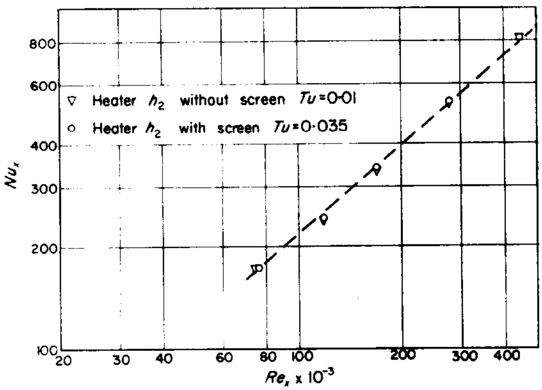

FIG. 24. Heat transfer from heater h_2; turbulent boundary layer.

3.4 *Turbulent heat transfer*

It is not difficult to guess that a fully developed turbulent boundary layer should be insensitive to changes in free-stream turbulence. In order to verify this hypothesis, experiments were undertaken with the same arrangement, except that the boundary layer was tripped at the leading edge *a* in Fig. 1. Tripping was achieved by mounting a strip of coarse emery paper 2 cm wide and a tripping rod 0·245 in dia. placed 0·5 in downstream from the leading edge.

The results of these measurements are shown in Figs. 23 and 24 in the form of plots of $Nu_x = hx/k$ against $Re_x = U(x) . x/\nu$, where x is measured from the tripping wire onwards. It is seen that a change in the intensity of turbuence from $Tu = 0{\cdot}01$ to $Tu = 0{\cdot}035$ or $Tu = 0{\cdot}045$ exercises no effect on the rate of heat transfer whatsoever.

As a matter of interest, we note that Nu_x is proportional to $Re_x^{0{\cdot}796}$ for h_1 and that Nu_x is proportional to $Re_x^{0{\cdot}858}$ for h_2.

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Résumé—Cet article décrit les résultats d'expériences sur les effets combinés d'un gradient de pression favorable et d'une intensité de turbulence dans l'écoulement libre sur le transport de chaleur local à partir d'une plaque isotherme plane vers un écoulement d'air. Des expériences ont été faites à la fois avec des couches limites laminaires et turbulentes.

Les expériences montrent qu'au-dessus d'une intensité de turbulence d'environ 1 pour cent, le flux local de convection laminaire est augmenté, l'augmentation elle-même croissant avec l'intensité de la turbulence. En contraste avec le cas du point d'arrêt, l'augmentation est de grandeur modeste, étant de l'ordre de 5 à 10 pour cent pour $Tu \approx 5$ pour cent.

La couche limite turbulente se trouve être insensible aux variations de l'intensité de la turbulence dans l'écoulement libre.

Une série d'enregistrements pris avec une sonde à fil chaud démontre qu'une couche limite laminaire porte en elle des fluctuations très violentes d'un caractère manifestement stable. L'apparition de fréquences élevées et une légère augmentation d'amplitude lorsque le nombre de Reynolds croît ont été relevées d'une façon qualitative.

Zusammenfassung—In der Arbeit werden Versuchsergebnisse beschrieben über den kombinierten Einfluss eines geeigneten Druckgradienten und einer Freistromturbulenz unterschiedlicher Intensität auf den örtlichen Wärmeübergang von einer isothermen ebenen Platte an einen Luftstrom. Die Versuche wurden sowohl bei laminarer als auch bei turbulenter Grenzschicht unternommen.

Es zeigt sich, dass über einer Turbulenzintensität von etwa 1% die örtliche, laminare Konvektion zunimmt, wobei sich die Zunahme selbst mit der Turbulenzintensität erhöht. Im Gegensatz zum Staupunkt ist die Zunahme bescheiden, nämlich nur in der Grössenordnung 5–10% bei $Tu \approx 5\%$.

Die turbulente Grenzschicht erwies sich als unempfindlich gegen Änderungen der Intensität des Freistroms.

Eine Reihe von Versuchen mit einer Hitzdrahtsonde zeigt, dass die laminare Grenzschicht ziemlich intensive Schwankungen von offenbar stabilem Charakter enthält. Das Auftreten höherer Frequenzen und eines leichten Anstiegs der Amplitude mit zunehmender Reynolds-Zahl wurde qualitativ untersucht.

Аннотация—В статье представлены результаты экспериментов по одновременному воздействию отрицательного градиента давления и интенсивности турбулентности потока на локальный теплообмен изотермической плоской пластины с потоком воздуха как при ламинарном, так и турбулентном режиме течения в пограничном слое.

Эксперименты показали, что при интенсивности турбулентности выше 1% локальная скорость ламинарной конвекции возрастает, причем это возрастание увеличивается с увеличением интенсивности турбулентности. По сравнению с точкой отрыва это возрастание имеет величину порядка 5–10% для $Tu \approx 5\%$.

Оказалось, что турбулентный пограничный слой нечувствителен к изменениям интенсивности турбулентности основного потока.

Серия диаграмм, сделанных при помощи нагретой проволочки, показывает, что ламинарный пограничный слой имеет довольно интенсивные пульсации очевидно устойчивого характера. Была получена качественная картина возникновения более высоких частот и слабого возрастания амплитуды при увеличении числа Рейнольдса.